REPORT 1011

DYNAMICS OF A TURBOJET ENGINE CONSIDERED AS A QUASI-STATIC SYSTEM[1]

By Edward W. Otto and Burt L. Taylor, III

SUMMARY

A determination of the dynamic characteristics of a typical turbojet engine with a centrifugal compressor, a sonic-flow turbine-nozzle diaphragm, and fixed-area exhaust nozzle is presented. A generalized equation for the transient behavior of the engine was developed; this equation was then verified by calculations using compressor- and turbine-performance charts extrapolated from equilibrium operating data and by experimental data obtained from an engine operated under transients in fuel flow.

The results indicate that a linear differential equation for engine acceleration as a function of fuel flow and engine speed for operation near a steady-state operating condition can be written. The coefficients of this equation can be obtained either from actual transient data or with a fair degree of accuracy from the steady-state performance maps of the compressor and turbine and can be corrected for altitude in the same manner that steady-state performance data are corrected.

INTRODUCTION

In a turbojet engine, the various engine variables such as speed and thrust respond comparatively slowly to changes in fuel flow—much more slowly than reciprocating-engine variables such as torque respond to changes in manifold pressure. Any control mechanism charged with the responsibility of keeping turbojet-engine variables at a set value is faced with the problem of changing the fuel flow in response to errors in the controlled variable at a rate that is fast enough to bring the variable back to the set value as quickly as possible but not so fast that excessive turbine temperatures are encountered. The same problem is encountered when the control is asked to change the speed or thrust from one set value to another. Thus, to insure the design of a successful control the designer must have information regarding the response rate of the particular engine variable that he is interested in controlling.

A study of the dynamic behavior of engine speed, made at the NACA Lewis laboratory in 1948, is presented herein. This variable rather than thrust has been studied because, in the turbojet engine, speed is almost analogous to thrust and can be more easily and more accurately measured. Obviously, other engine variables may possess as many advantages for use as a thrust-control parameter. A previous investigation at this laboratory has indicated, however, that speed is the best parameter available because no other engine variable exhibits as many advantages as does speed. At the present time speed is used almost universally as a thrust-control parameter in turbojet engines.

It was felt that the needs of the control designer would be best served if the dynamic behavior of engine speed could be expressed in mathematical form and if altitude corrections to this expression could be indicated. Also it is desirable to know whether or not the engine dynamics can be predicted within reason from the steady-state performance charts of the engine components. Accordingly, the study contained herein was pursued as follows: A differential equation relating speed and fuel flow was derived by considering the thermodynamic and flow processes to be quasi-static. Altitude corrections were then applied to this equation in the same manner as steady-state altitude corrections are applied. The coefficients of the terms of the differential equation were obtained by plotting accelerating torque as a function of fuel flow and speed from calculations involving the use of the steady-state performance charts of the engine components. A typical engine was then operated under certain transients of fuel flow to obtain experimental data for a verification of both the differential equation derived and the coefficients obtained from steady-state component data.

ANALYSIS

In a turbojet engine of the type under consideration, an increase in fuel flow at constant speed causes an increase in turbine-inlet temperature, which results in an increase in turbine torque. The difference between the turbine torque and the torque absorbed by the compressor then accelerates the engine according to the following equation:

$$\alpha=\frac{Q}{I}$$

[1] Supersedes NACA TN 2091, "Dynamics of Turbojet Engine Considered as Quasi-Static System" by Edward W. Otto and Burt L. Taylor, III, 1950.

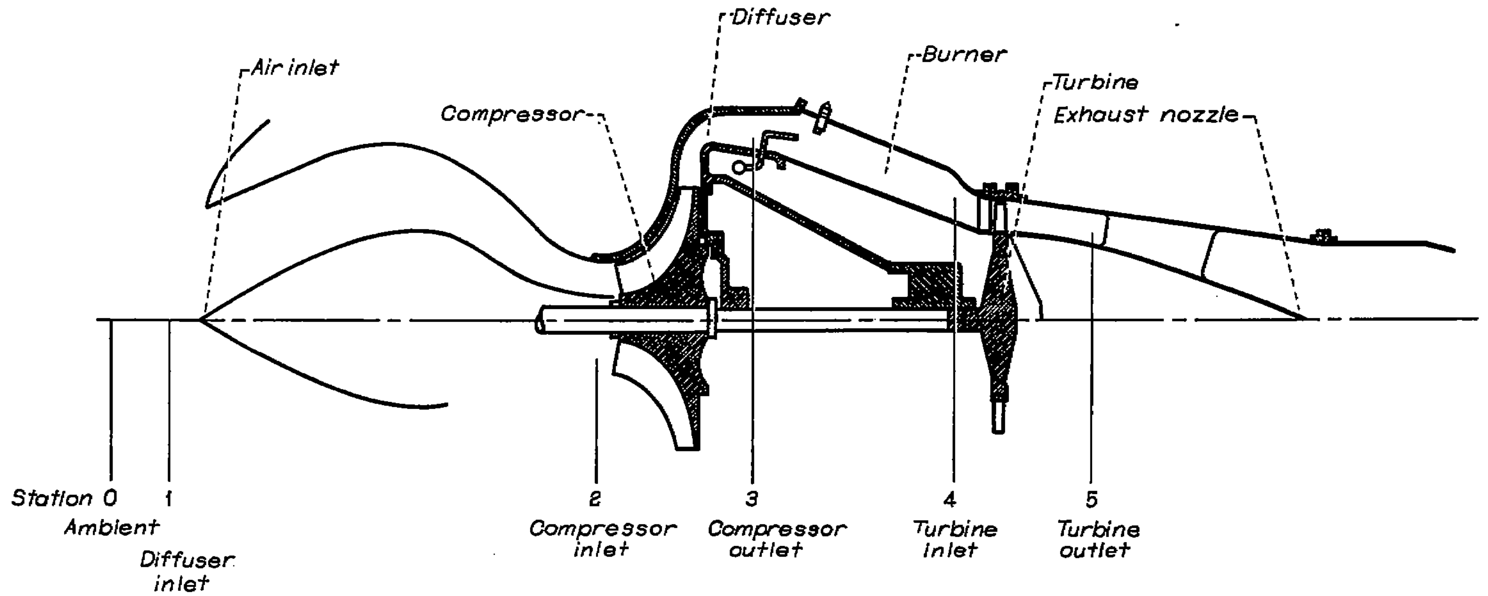


FIGURE 1.—Typical configuration of turbojet engine.

where α is the angular acceleration, Q is the difference between the instantaneous torque output of the turbine and the instantaneous torque absorbed by the compressor, and I is the polar moment of inertia of all rotating parts. Thus the effect of a change in fuel flow on engine acceleration is proportional to the effect of a change in fuel flow on the difference between torque output of the turbine and torque absorbed by the compressor. From a general consideration of the mechanics of the system, therefore, a differential equation relating speed and fuel flow can be written. This equation can then be correlated with data from a typical engine of the type under consideration. This procedure is demonstrated with an engine consisting of the components shown in figure 1.

DEVELOPMENT OF GENERALIZED ENGINE EQUATION

For the equilibrium running conditions of turbojet engines, various engine parameters may be represented by general functional forms of the one independent variable, effective fuel flow; that is, $N=f(W_f\eta_b)$, $T_4=g(W_f\eta_b)$, and so forth. (All symbols are defined in appendix A.) The development of general functional forms relating the variables during transient conditions of operation requires some hypothesis concerning the thermodynamic and flow processes during these periods. The hypothesis made herein is that the processes are quasi-static; that is, they act like a continuous series of equilibrium states. Each point of the dynamic state is an equilibrium condition for the flow and thermodynamic processes. It follows that a point reached by each component of the system during the transient corresponds to some equilibrium point for that component. Furthermore, the point encountered by the complete engine, being a sum of the components, is also some equilibrium point for the complete engine.

Without altering the turbojet-engine geometry, equilibrium operating points other than the equilibrium operating conditions can be obtained only by applying an outside load to the engine. The action of an external load creates an additional independent variable for the dynamic state. This action may be expressed in terms of some engine variable, which allows this variable to be considered as the additional independent variable. For transient conditions of operation, various dependent variables of the engine may therefore be considered as functions of the effective fuel flow and another engine variable.

Because this analysis is concerned with the transient behavior of engine speed, this variable was chosen as the additional independent variable for the dynamic state. Unbalanced torque was chosen as the dependent variable because of its relation to engine speed. The assumed functional relation is therefore:

$$Q=f(W_f\eta_b,N) \tag{1}$$

This function may be expanded about an initial point i as follows:

$$\begin{aligned} Q=f(W_f\eta_b,N)=Q_i+&\left[\frac{\partial Q}{\partial(W_f\eta_b)}\right]_i[W_f\eta_b-(W_f\eta_b)_i]+\\ &\left(\frac{\partial Q}{\partial N}\right)_i(N-N_i)+\frac{\left[\frac{\partial^2 Q}{\partial(W_f\eta_b)^2}\right]_i[W_f\eta_b-(W_f\eta_b)_i]^2}{2!}+\\ &\frac{2\left[\frac{\partial^2 Q}{\partial(W_f\eta_b)\,\partial N}\right]_i[W_f\eta_b-(W_f\eta_b)_i](N-N_i)}{2!}+\\ &\frac{\left(\frac{\partial^2 Q}{\partial N^2}\right)_i(N-N_i)^2}{2!}+\cdots \end{aligned} \tag{2}$$

When the deviation from the point i is sufficiently small, the terms containing products of higher derivatives and products or powers of small numbers become negligible and only the

first three terms of equation (2) are of significant value. Furthermore, if the point i is a steady-state point o, the initial value of accelerating torque Q_i is zero.

When the deviations from the steady state are denoted by Δ, the significant terms of equation (2) can be written as

$$Q=\left[\frac{\partial Q}{\partial(W_f\eta_b)}\right]_o \Delta(W_f\eta_b)+\left(\frac{\partial Q}{\partial N}\right)_o \Delta N \tag{3}$$

The accelerating torque can also be expressed as a function of engine acceleration as

$$Q=\frac{I\,d(N-N_o)}{dt}=I\,\frac{d(\Delta N)}{dt} \tag{4}$$

Substitution of equation (4) in equation (3) and rearrangement give

$$I\,\frac{d(\Delta N)}{dt}=\left[\frac{\partial Q}{\partial(W_f\eta_b)}\right]_o \Delta(W_f\eta_b)+\left(\frac{\partial Q}{\partial N}\right)_o \Delta N \tag{5}$$

Equation (5) represents the dynamic behavior of engine speed in that it gives engine acceleration as a function of fuel flow and speed.

A graphical representation of equation (5) for given altitude and ram conditions is shown in figure 2 (a). The $Q=0$ line is the steady-state operating line. The slope of any constant-speed line of figure 2(a) at $Q=0$ is $\left[\frac{\partial Q}{\partial(W_f\eta_b)}\right]_o$ and the slope of any constant-fuel-flow line of figure 2 (a) at $Q=0$ is $\left(\frac{\partial Q}{\partial N}\right)_o$. Equation (5) can therefore be rewritten as

$$I\,\frac{d(\Delta N)}{dt}=a_o\Delta(W_f\eta_b)-b_o\Delta N \tag{6}$$

where a_o is the numerical value of the slope of the torque-fuel-flow curves at constant speed and b_o is the numerical value of the slope of the torque-speed curves at constant fuel flow.

Equation (6) holds only for constant altitude and ram conditions. Because speed, fuel-flow, and torque can all be corrected for altitude, however, equation (6) can be generalized to be true for all altitude conditions. By applying the standard corrections to the speed, fuel-flow, and torque terms in equation (3), this equation can be rewritten in terms of corrected quantities:

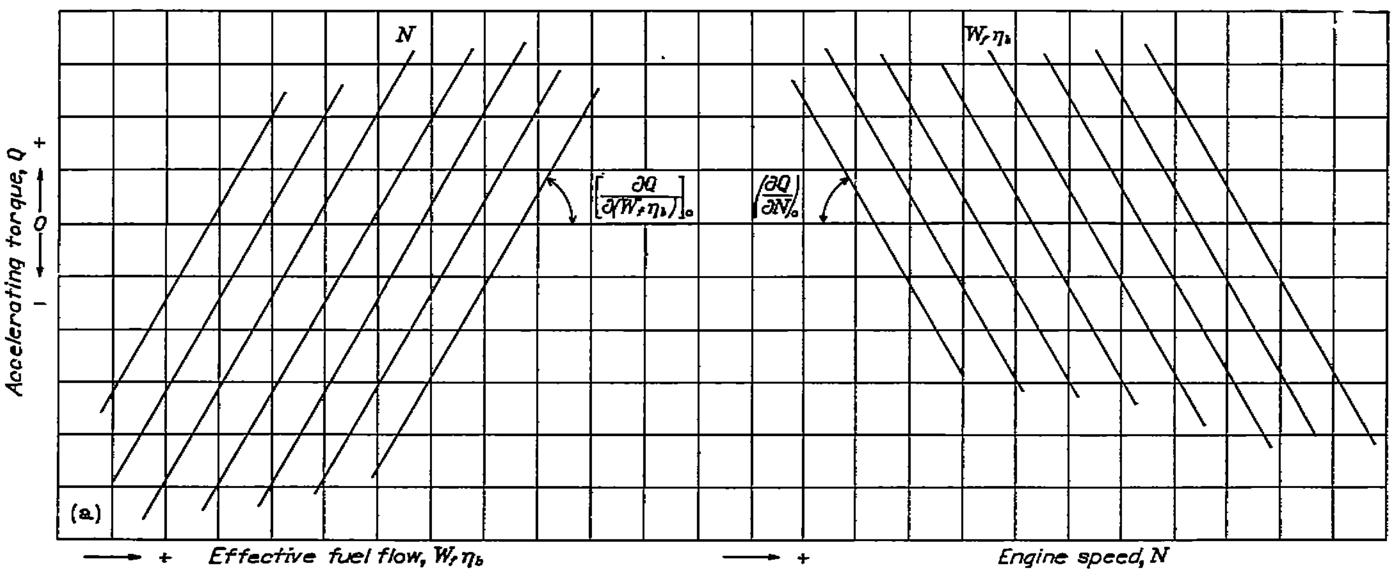
$$\frac{Q}{\delta_2}=\left[\frac{\partial\left(\frac{Q}{\delta_2}\right)}{\partial\left(\frac{W_f\eta_b}{\delta_2\sqrt{\theta_2}}\right)}\right]_o [W_f\eta_b-(W_f\eta_b)_o]\,\frac{1}{\delta_2\sqrt{\theta_2}}+\left[\frac{\partial\left(\frac{Q}{\delta_2}\right)}{\partial\left(\frac{N}{\sqrt{\theta_2}}\right)}\right]_o (N-N_o)\,\frac{1}{\sqrt{\theta_2}} \tag{7}$$

Equation (4) may be written as follows after both sides of the equation are divided by δ_2 and $\sqrt{\theta_2}$ and terms are collected:

$$\frac{Q}{\delta_2}=\frac{I\sqrt{\theta_2}}{\delta_2}\,\frac{d}{dt}\left(\frac{\Delta N}{\sqrt{\theta_2}}\right) \tag{8}$$

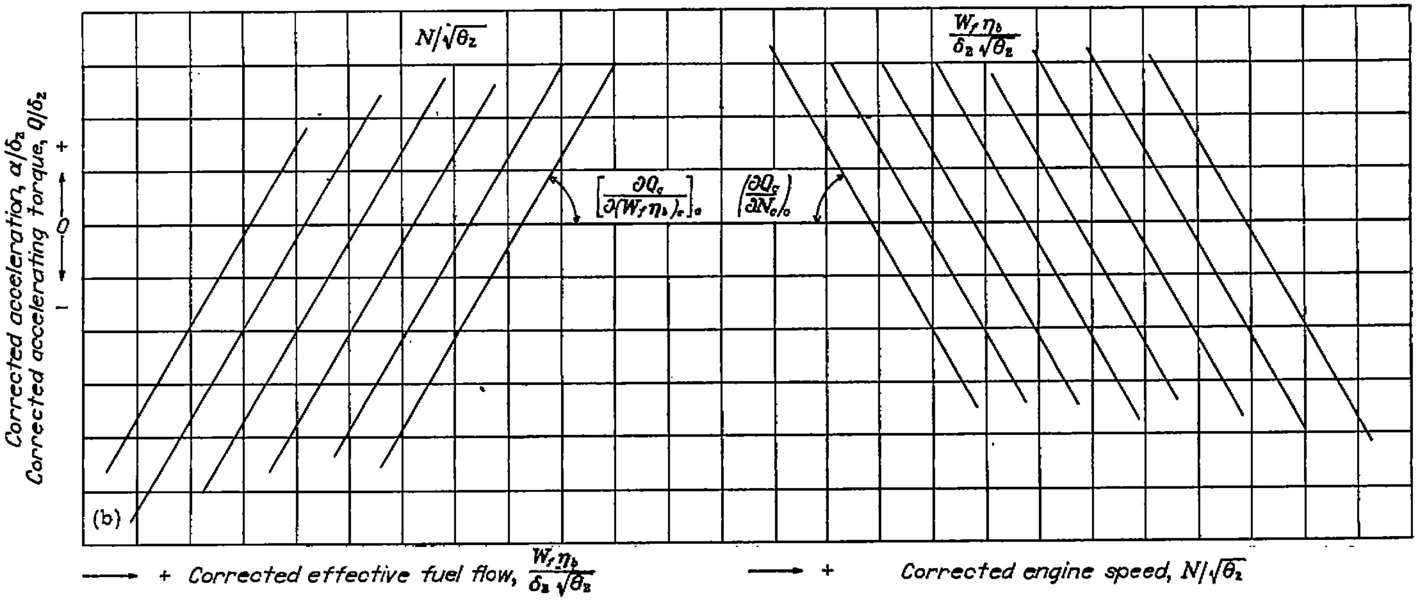
When equation (8) is substituted in equation (7) the following equation results:

$$\frac{I\sqrt{\theta_2}}{\delta_2}\,\frac{d}{dt}\left(\frac{\Delta N}{\sqrt{\theta_2}}\right)=\left[\frac{\partial\left(\frac{Q}{\delta_2}\right)}{\partial\left(\frac{W_f\eta_b}{\delta_2\sqrt{\theta_2}}\right)}\right]_o \frac{\Delta(W_f\eta_b)}{\delta_2\sqrt{\theta_2}}+\left[\frac{\partial\left(\frac{Q}{\delta_2}\right)}{\partial\left(\frac{N}{\sqrt{\theta_2}}\right)}\right]_o \frac{\Delta N}{\sqrt{\theta_2}} \tag{9}$$



(a) Engine variables uncorrected.

FIGURE 2.—Graphical representation of engine differential equation for condition of constant partials.



(b) Engine variables corrected to compressor-inlet conditions relative to NACA standard atmospheric conditions at sea level.

FIGURE 2.—Concluded. Graphical representation of engine differential equation for condition of constant partials.

By designating all the corrected quantities with a subscript c and by substituting the corrected curve slopes (fig. 2 (b)) for the partials, equation (9) may be written as

$$\frac{I\sqrt{\theta_2}}{\delta_2}\frac{d(\Delta N_c)}{dt}=a_{o,c}\Delta(W_f\eta_b)_c-b_{o,c}\Delta N_c \qquad (10)$$

Equation (10) represents the dynamic behavior of the engine in terms of corrected engine variables at constant ram.

An examination of equation (10) reveals several pertinent facts concerning the behavior of engine acceleration under altitude conditions. First, the coefficients $a_{o,c}$ and $b_{o,c}$ are corrected quantities, which are nearly independent of altitude although they do depend on ram. Second, the burner efficiency affects the dynamic behavior of the engine. Burner efficiency may have a considerable effect on engine acceleration because the efficiency may change substantially under transients in fuel flow. Last, equation (10) indicates that the effective moment of inertia $I\sqrt{\theta_2}/\delta_2$ increases with increasing altitude. Thus the permissible termperature-limited acceleration is reduced with increasing altitude.

Equation (10) may be rearranged into the usual form for a differential equation describing a simple first-order lag system:

$$\frac{I\sqrt{\theta_2}}{\delta_2 b_{o,c}}\frac{d(\Delta N_c)}{dt}+\Delta N_c=\frac{a_{o,c}}{b_{o,c}}\Delta(W_f\eta_b)_c \qquad (11)$$

With the equation written in this form, the coefficient of the derivative term is the time constant τ for the system. Equation (11) shows that the effect of increased altitude is to increase the engine time constant above its sea-level value by the factor $\sqrt{\theta_2}/\delta_2$.

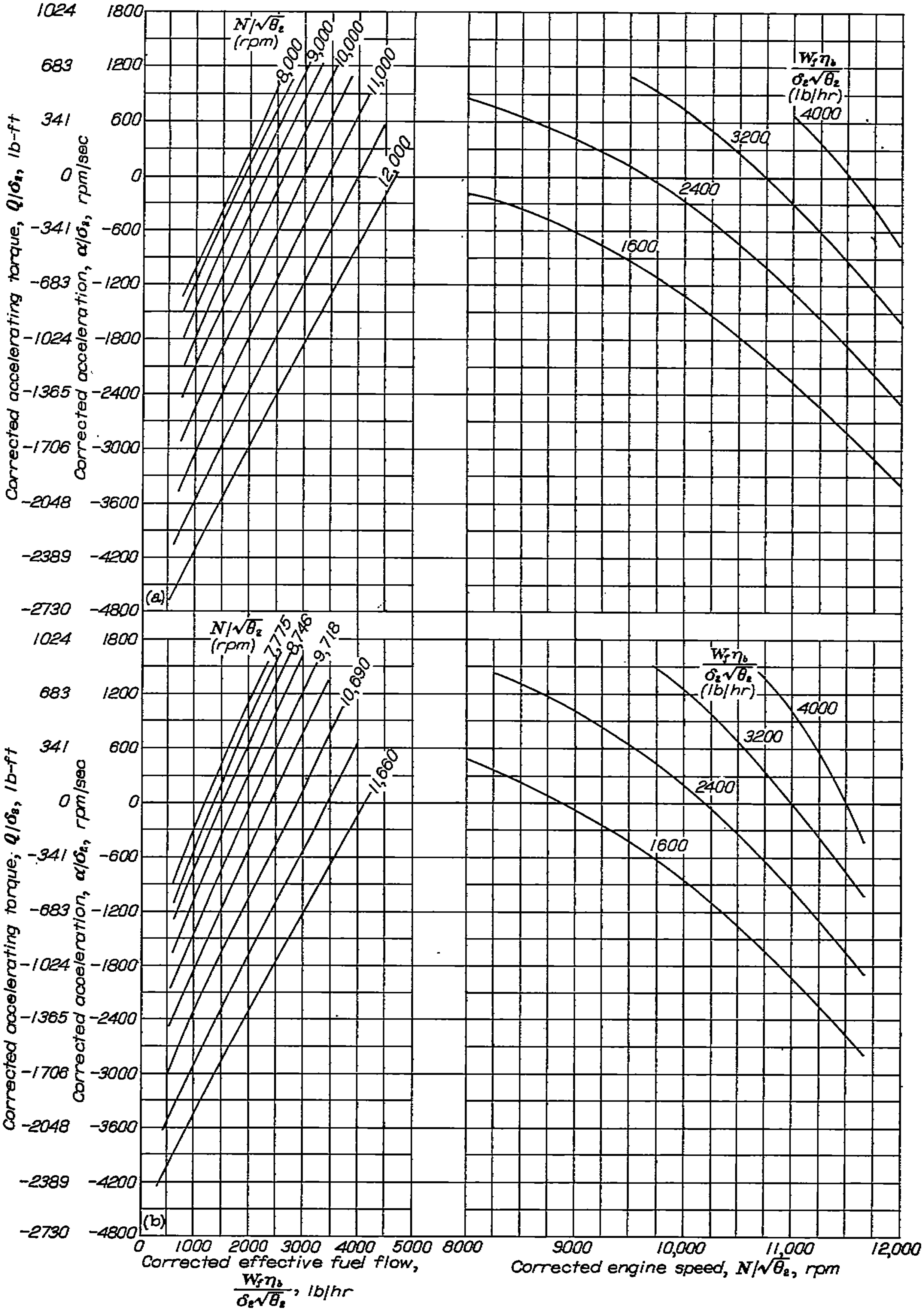
EVALUATION OF ENGINE CONSTANTS FROM STEADY-STATE CHARACTERISTICS

In order to evaluate the slopes a_o and b_o, calculations were made for a typical turbojet engine with a centrifugal compressor, a sonic-flow turbine-nozzle diaphragm, and a fixed-area exhaust nozzle by use of the compressor-performance charts and the thermodynamic relations in the engine. The method of making these calculations is presented in appendix B and is briefly outlined as follows: At constant ambient conditions, the compressor torque and the turbine torque were determined for a number of combinations of the parameters, turbine-inlet temperature and engine speed—parameters that are common to both the compressor and the turbine in a direct-coupled engine. The calculations were made by assuming constant compressor slip factor, burner efficiency, fuel-air ratio, burner pressure ratio, turbine efficiency, nozzle efficiency, and percentage accessory and gear loss. The difference in torque between the compressor and the turbine at any speed was considered an accelerating torque, and the parameter turbine-inlet temperature was expressed in terms of fuel flow. From these data, acceleration could be plotted in terms of engine speed and fuel flow for a set of constant ambient conditions. Because all the engine parameters may be corrected for altitude, the calculations were carried through in terms of corrected quantities.

The results of these calculations for two ram conditions corresponding to 0 and 340 miles per hour at sea level are shown in figure 3. The curves of the left-hand plot are essentially straight lines with variable spacing that cross-plot to the curved lines shown on the right-hand side. The curves of the left-hand side are nearly straight lines because of the constant efficiencies assumed in the calculations. If variable efficiencies are assumed, the shape of the curves will be changed depending on the variation of efficiencies assumed.

For the constant efficiencies assumed, the slope a_o is nearly constant over the power range and for large deviations from the steady-state region. The slope b_o, however, changes substantially over the power range and for large deviations from the steady-state region. This change in b_o indicates that some of the terms dropped from the expansion (equation (1)) may have significant value, especially for large deviations from the steady-state line.

Because the product of the moment of inertia and the reciprocal of b_e is the engine time constant, figure 3 indicates that the engine time constant depends to a considerable extent on the operating point of the engine, which is roughly defined by the engine speed and altitude. The effect of ram on both a_e and b_e is rather small.



(a) Ram pressure ratio, 1.0. (b) Ram pressure ratio, 1.2.

FIGURE 3.—Relation among acceleration or accelerating torque, effective fuel flow, and engine speed corrected to compressor-inlet conditions relative to NACA standard atmospheric conditions at sea level as calculated from steady-state component data.

EVALUATION OF ENGINE CONSTANTS FROM TRANSIENT DATA

In order to determine the validity of the preceding analysis, an engine of the type for which the calculations were made was operated under conditions in which the fuel flow was rapidly changed through a series of magnitudes in order to obtain as large a region as possible of nonequilibrium operation. In attempting to impose operation of this nature on the engine, it was necessary to include components in the fuel system by means of which the fuel flow could be rapidly changed from one value to another. The system used consisted of two fuel lines from the pump to the fuel manifold arranged in parallel. The regular engine throttle was installed in one line and another engine throttle in series with a solenoid valve was installed in the other line. With the solenoid valve closed, the regular engine throttle was adjusted to give a selected lower engine speed. With the solenoid valve open, the auxiliary throttle was adjusted to give a selected higher engine speed. The fuel flows for any two steady-state engine speeds could thus be established and nonequilibrium conditions were obtained by opening or closing the solenoid valve.

Fuel flow was measured by means of a thin-plate orifice in the main fuel line with a differential-pressure gage to measure the pressure drop across the orifice. Engine speed was measured by a standard electric tachometer indicating revolutions per minute and time was measured by a clock reading in thousandths of a minute. The data were recorded simultaneously by a motion-picture camera at the rate of 12 frames per second. A preliminary investigation of the response rate of the fuel-flow and speed-measuring instrumentation showed that it was capable of following much more rapid rates of change than those encountered during the engine runs.

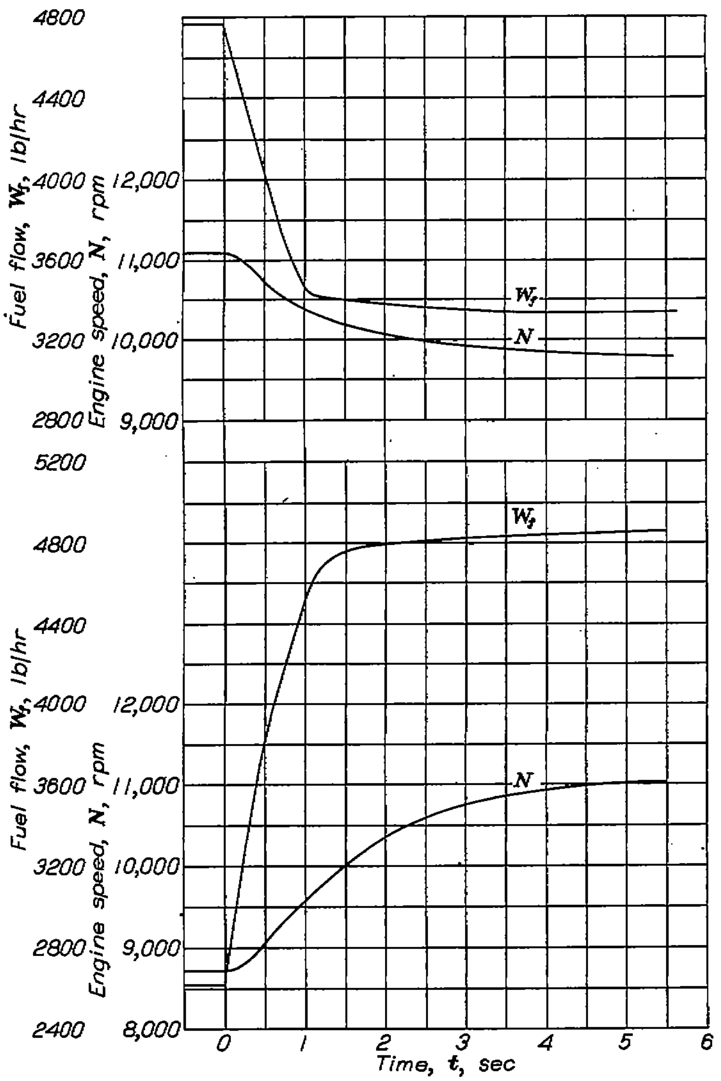


FIGURE 4.—Typical time-domain curves of fuel flow and engine speed for engine transient experiments.

Typical examples of the imposed transients of fuel flow for accelerations and decelerations together with the resulting speed-time curves are shown in figure 4. The experimental data are shown in figure 5 in the same form as the calculated data in figure 3 except that, for the experimental data, the actual fuel flow is used instead of the effective fuel flow. The left-hand side of figure 5 was obtained from the data as follows: At a given speed, the acceleration and the fuel flow corresponding to the same instant in time were measured. This procedure was repeated for a number of transient runs resulting from various magnitudes of fuel-flow change. The right half of figure 5 was obtained by cross-plotting the faired curves of the left-hand side. The grouping of the data points from several runs indicates that a unique relation exists among torque, fuel flow, and speed, regardless of the manner in which the point is reached.

Near the steady-state region, the experimental data follow the results of the analysis and the calculations in that the slope $a_{o,c}$ is virtually constant over the power range and the slope $b_{o,c}$ varies substantially over the power range as indicated in figure 5. For accelerations in excess of about ± 5 percent of rated speed per second, the engine begins to exhibit large deviations from the calculated curves.

EFFECT OF ENGINE CONSTANTS ON CONTROLLED ENGINE

In matching controls to engines, the damping ratio is one of the criterions used to evaluate the degree of matching. Both engine coefficients and control coefficients appear in this damping term. For example, if it is assumed that a control operating on speed error to vary fuel flow according to the following equation

$$W_f=\int \beta(N_s-N)\,dt+\epsilon(N_s-N)+\sigma\frac{d(N_s-N)}{dt}$$

is applied to the engine described by equation (11), the combined equation for the system becomes

$$\left(\frac{I\theta_2}{a_{o,c}\eta_b}+\sigma\sqrt{\theta_2}\right)N_c''+\left(\frac{b_{o,c}\,\delta_2\sqrt{\theta_2}}{a_{o,c}\,\eta_b}+\epsilon\sqrt{\theta_2}\right)N_c'+$$

$$\beta\sqrt{\theta_2}N_c=\beta N_s+\epsilon N_s'+\sigma N_s''$$

where the primes signify derivatives with respect to time. The damping ratio for this expression is

$$\frac{\dfrac{b_{o,c}\,\delta_2}{a_{o,c}\eta_b}+\epsilon}{2\sqrt{\beta\left(\dfrac{I\sqrt{\theta_2}}{a_{o,c}\,\eta_b}+\sigma\right)}} \tag{12}$$

Thus if it is desired to maintain a constant value of the damping term, such as critical damping, it is evident from an examination of equation (12) that if an engine coefficient varies, some control coefficient must be varied in such a

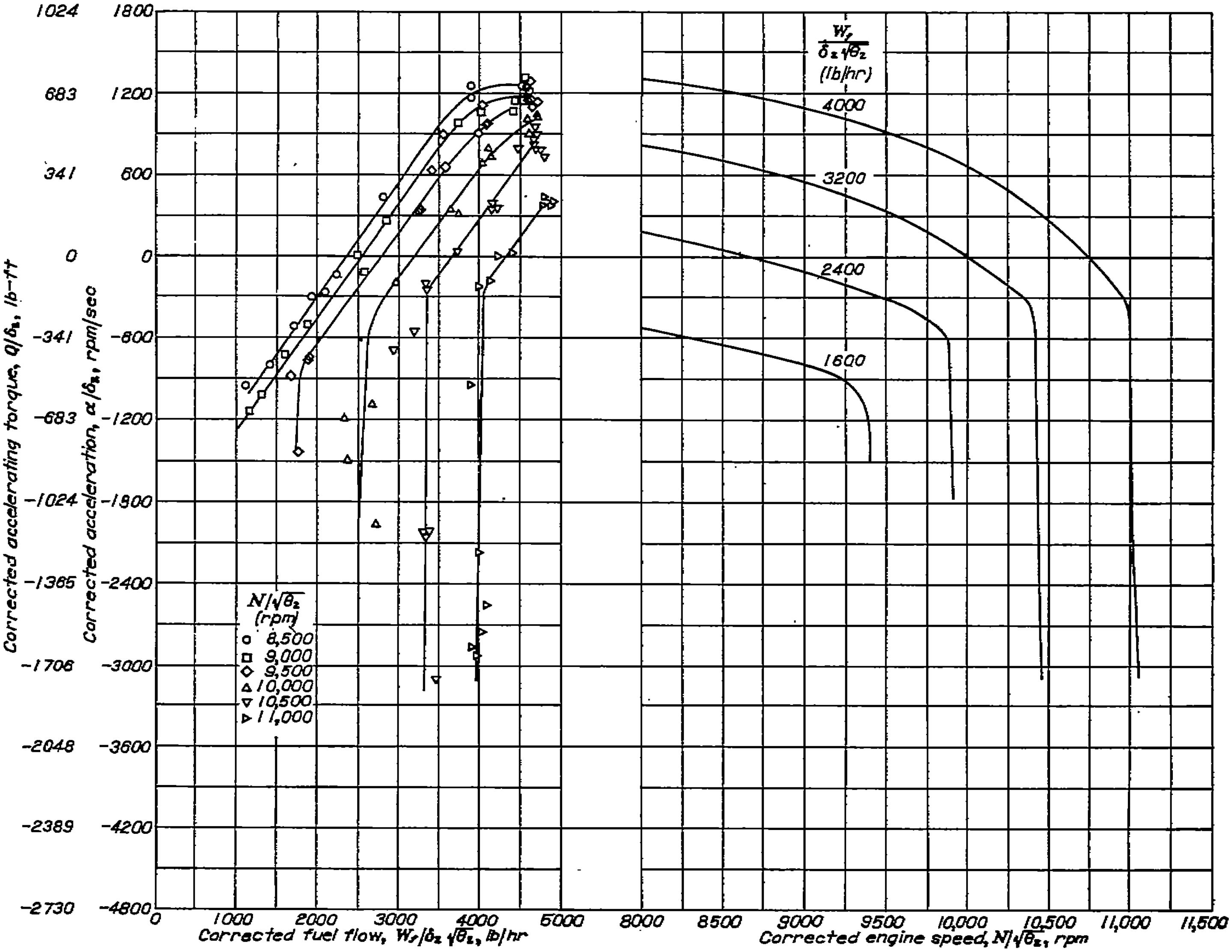


FIGURE 5.—Relation among acceleration or accelerating torque, fuel flow, and engine speed corrected to compressor-inlet conditions relative to NACA standard atmospheric conditions at sea level as obtained from experimental data for a ram pressure ratio of 1.0

manner that the damping ratio remains substantially constant. Figures 6 and 7 are plots of $a_{o,c}$ and $b_{o,c}$, respectively, as functions of engine speed for the calculated data and the experimental data. The slope $a_{o,c}$ is virtually constant and will have little effect on the damping ratio but $b_{o,c}$ may change the damping ratio considerably for various operating points. If the variation is considered too great, it will be necessary to vary a control coefficient to compensate for the variation in $b_{o,c}$. Figure 7 indicates that $b_{o,c}$ varies almost directly with engine speed, which would indicate that one method of compensating for the variation of $b_{o,c}$ is to vary a control coefficient as a function of engine speed.

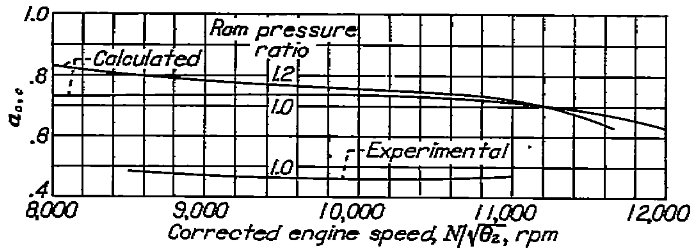


FIGURE 6.—Values of partial of torque with respect to fuel flow or effective fuel flow at steady-state point plotted against engine speed corrected to compressor-inlet conditions relative to NACA standard atmospheric conditions at sea level for calculated and experimental data.

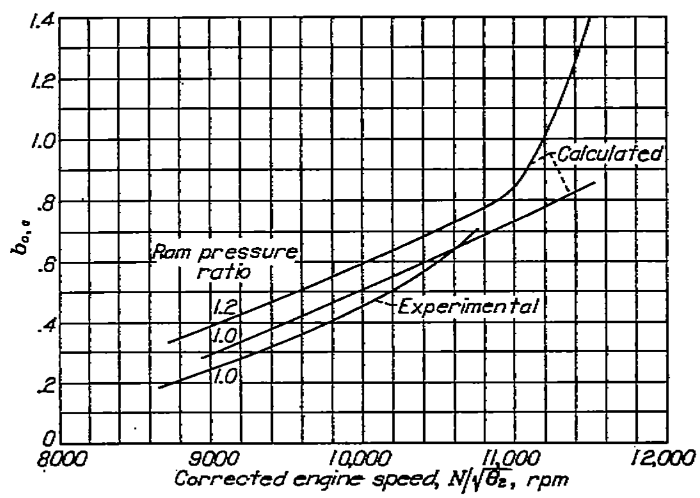


FIGURE 7.—Values of partial of torque with respect to engine speed at steady-state point plotted against engine speed corrected to compressor-inlet conditions relative to NACA standard atmospheric conditions at sea level for calculated and experimental data.

A plot of the corrected engine time constant as a function of engine speed is shown in figure 8. The time constant for this engine varies through a 4 to 1 range over the power range of the engine.

DISCUSSION

In general, the correlation between the calculated results and the experimental results is considered good and would be better if an approximation of burner efficiency were included in the calculations. This correlation indicates that for an engine of the type investigated, operating near the steady-state region, a good approximation of the dynamic characteristics can be made as soon as the component characteristics are available. Thus an engine may be

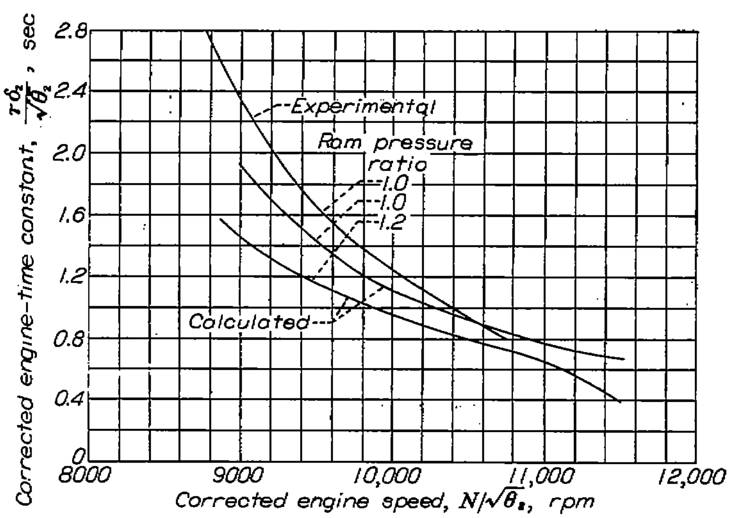


FIGURE 8.—Values of corrected engine time constant at steady-state point plotted against engine speed corrected to compressor-inlet conditions relative to NACA standard atmospheric conditions at sea level for calculated and experimental data.

designed, at least to a certain extent, to have good transient characteristics as well as good steady-state performance characteristics or at least a compromise may be reached between transient and steady-state performance if the two conflict in certain configurations.

Although the correlation in the steady-state regions is good, the correlation in the region of large accelerations can be regarded as only fair and in the region of large decelerations as poor. A possible explanation for the poorer correlation in the accelerating region is that the engine is approaching the acceleration blow-out region in the burners, which is a region of apparently low burner efficiencies. Temperature data taken during the experiments indicated that when excessive amounts of fuel were suddenly added combustion occurred through the turbine and the tail cone, which would result in low burner efficiencies during these periods.

The large deviations between calculated and observed values of deceleration seem to have no obvious explanation. The very large decelerations observed would indicate that very large reductions in torque output of the turbine occurred. These reductions can be only partly accounted for by the normal steady-state effect of a reduction in temperature on turbine torque. The reduction in temperature that did occur may possibly have caused a rotation of the turbine-entrance-velocity vector relative to the blade to a condition of negative angle of attack. Such a condition corresponds to a high value of turbine-velocity ratio for which the turbine efficiency is rapidly falling. The existence of a negative angle of attack together with the increased air flow and consequently increased compressor torque that results from a reduction in temperature at constant speed could conceivably cause large decelerating torques. The existence of a negative angle of attack would also explain the position of the sharp breaks in the curves of figure 5, because at low speeds the steady-state angle of attack is greater than at high speeds, which are near the design condition; therefore a greater reduction in temperature at constant speed is necessary to cause negative angles of attack and a consequent sharp change in curve slope. The preceding discussion would seem to indicate a possible explanation for the sharp change of slope for deceleration points but does not explain the vertical parts of the curves, which indicate that various decelerations occur with the same turbine-inlet temperature. A check of the data, however, indicated that along the vertical part of a constant-speed line of figure 5 the deceleration value was consistent with the rate of change of fuel flow; that is, the deceleration value increased with increasing rates of change of fuel flow. Thus, if the burner efficiency reduced as a function of the rate of change of fuel flow so that the temperature was reduced, the vertical lines of figure 5 seem logical.

CONCLUSIONS

The following conclusions are drawn from an analysis of the dynamic behavior of, and from transient data obtained from, a typical turbojet engine with a centrifugal compressor, a sonic-flow turbine-nozzle diaphragm, and a fixed-area exhaust nozzle:

1. Accelerating torque is a function of fuel flow and engine speed. A linear differential equation approximately represents the transient behavior of the engine speed for accelerations less than approximately 5 percent of rated speed per second.

2. The transient equation of the engine may be obtained from the steady-state performance of the engine components to a degree of accuracy that will permit the prediction of the engine transient characteristics as soon as the component characteristics are available.

3. A transient equation for engine speed may be derived including the effects of altitude by the use of corrected engine variables in the derivation.

4. The manner in which the control constants should be varied to compensate for the changes in engine characteristics with altitude and with engine speed may be predicted from an analysis of the combined engine and control equation.

LEWIS FLIGHT PROPULSION LABORATORY,
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
CLEVELAND, OHIO, *July 27, 1949.*

APPENDIX A

SYMBOLS

The following symbols have been used throughout this report:

A	exhaust-nozzle area, sq ft
a	partial of torque with respect to either effective or actual fuel flow at constant engine speed $\left(\frac{\partial Q}{\partial(W_f\eta_b)} \text{ or } \frac{\partial Q}{\partial W_f}\right)$
b	partial of torque with respect to engine speed at constant values of either actual or effective fuel flow $\left(\frac{\partial Q}{\partial N}\right)$
$c_{p,C}$	average specific heat for gas passing through compressor (assumed at 0.243 Btu/ (lb) (°F))
$c_{p,T}$	average specific heat for gas passing through turbine (assumed at 0.276 Btu/ (lb) (°F))
H_f	lower heating value of fuel, 18,400 Btu/lb
h	enthalpy, Btu/lb
I	polar moment of inertia of rotating parts, $5.427\left(\frac{2\pi}{60}\right)$ (lb-ft) (sec) (min) (rad)/revolution
N	engine speed, rpm
N_s	set value of engine speed, rpm
P	total pressure, lb/sq ft
p	static pressure, lb/sq ft
Q	accelerating torque, lb-ft
Q_C	torque required by compressor, lb-ft
Q_T	torque output of turbine, lb-ft
T	total temperature, °R
t	time, sec
W_a	air flow, lb/sec
W_f	fuel flow, lb/hr
W_g	total gas flow, lb/sec
α	acceleration of engine speed, rpm/sec
β	coefficient of integral control component, $\frac{\text{lb/hr}}{\text{(rpm) (sec)}}$
γ	ratio of specific heats
δ_2	altitude pressure ratio, $P_2/14.7$
ϵ	coefficient of proportional control component, $\frac{\text{lb/hr}}{\text{rpm}}$
η_b	burner efficiency, percent
η_T	turbine efficiency (assumed at 83 percent), percent
θ_2	altitude temperature ratio, $T_2/518.6$
σ	coefficient of derivative control component, $\frac{\text{(lb/hr) sec}}{\text{rpm}}$
τ	engine time constant, sec

Subscripts:

c	corrected
i	any engine operating point
o	any steady-state operating point
0	ambient
2	compressor inlet
3	compressor outlet
4	turbine inlet
5	turbine outlet

APPENDIX B

ENGINE-TRANSIENT-CHARACTERISTIC CALCULATIONS

In general, the method used to calculate the characteristics of an engine operating under transient conditions involves determining the difference between compressor torque and turbine torque for assumed values of engine speed, turbine-inlet temperature, ambient conditions, and ram pressure ratio. Figure 9 is a chart of typical compressor characteristics obtained from unpublished data. For assumed values of ram pressure ratio, ambient conditions, engine speed, and turbine-inlet temperature, corresponding values of compressor pressure ratio and corrected air flow may be determined. The torque required by the compressor for the assumed conditions may then be calculated from the following equations:

$$\frac{T_3}{T_2}=1+0.5079\times10^{-8}\left(\frac{N}{\sqrt{\theta_2}}\right)^2 \tag{B1}$$

$$Q_C=c_{p,C}(T_3-T_2)\frac{W_a(778)(60)}{2\pi N} \tag{B2}$$

Equation (B1) is developed for an assumed slip factor of 0.925 and a compressor rotor diameter of 2.5 feet. The definitions of the symbols used are given in appendix A.

The equation for the torque output of the turbine is similar to equation (B2) in that it involves the temperature drop through the turbine, the gas flow, and the speed. The speed and the gas flow are known from the assumed conditions and the resulting compressor point. The temperature drop through the turbine, however, must be determined for the operating point. In the absence of a complete turbine-performance map, general thermodynamic relations defining typical turbine operation must be utilized. A relation between turbine pressure ratio P_4/P_5 and turbine temperature ratio $\sqrt{T_4/T_5}$ is determined by assuming a constant turbine efficiency as expressed by the following equation:

$$\eta_T=\frac{1-\frac{T_5}{T_4}}{1-\left(\frac{P_5}{P_4}\right)^{\frac{\gamma_4-1}{\gamma_4}}} \tag{B3}$$

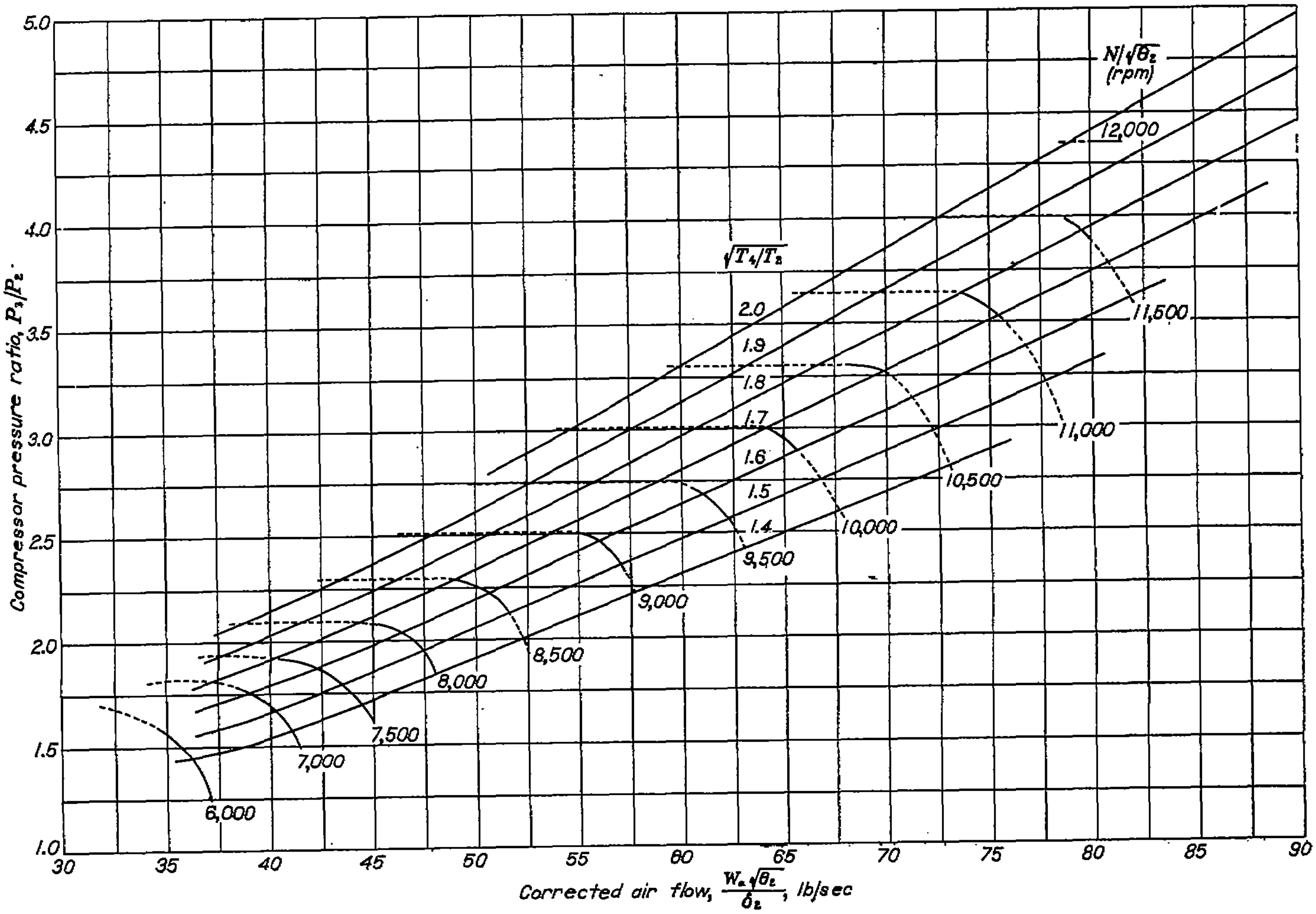


FIGURE 9.—Compressor characteristics corrected to compressor-inlet temperature and pressure relative to NACA standard atmospheric conditions at sea level. Curves of constant ratio of turbine-inlet temperature to compressor-inlet temperature for engine are superimposed on compressor-characteristic curves. (Obtained from unpublished data.)

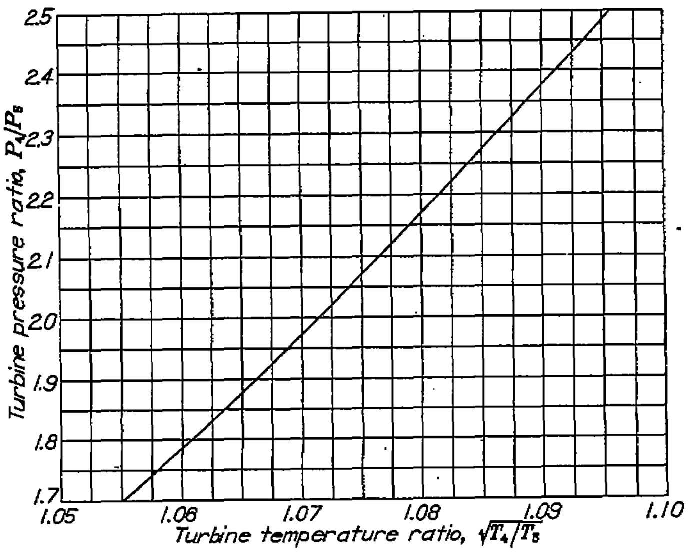


FIGURE 10.—Relation between turbine total-pressure ratio and total-temperature ratio at constant turbine efficiency of 0.83.

For this analysis η_T was assumed at 83 percent, and γ_4 was assumed at 1.33. The resulting relation between $\sqrt{T_4/T_5}$ and P_4/P_5 is plotted in figure 10.

Because critical flow exists in the turbine nozzles over the power range, the corrected gas flow through the turbine nozzles will have a constant value. For this engine, unpublished data have indicated that the corrected gas flow will have a value expressed by the following equation:

$$\frac{W_g\sqrt{\left(\frac{T_4}{519}\right)\left(\frac{\gamma_4}{1.4}\right)}}{\left(\frac{P_4}{14.7}\right)\left(\frac{\gamma_4}{1.4}\right)}=41.6 \text{ lb/sec} \tag{B4}$$

This equation may be expanded as follows:

$$\frac{W_g\sqrt{\frac{T_5}{519}}}{A\,\frac{P_5}{14.7}}=\frac{41.6\sqrt{\frac{\gamma_4}{1.4}}\left(\frac{P_4}{P_5}\right)}{A\sqrt{\frac{T_4}{T_5}}}$$

For an effective exhaust-nozzle area of 1.87 square feet and with γ_4 assumed constant at 1.33, the preceding equation becomes

$$\frac{W_g\sqrt{\theta_5}}{A\delta_5}=\frac{21.75\,\frac{P_4}{P_5}}{\sqrt{\frac{T_4}{T_5}}} \tag{B5}$$

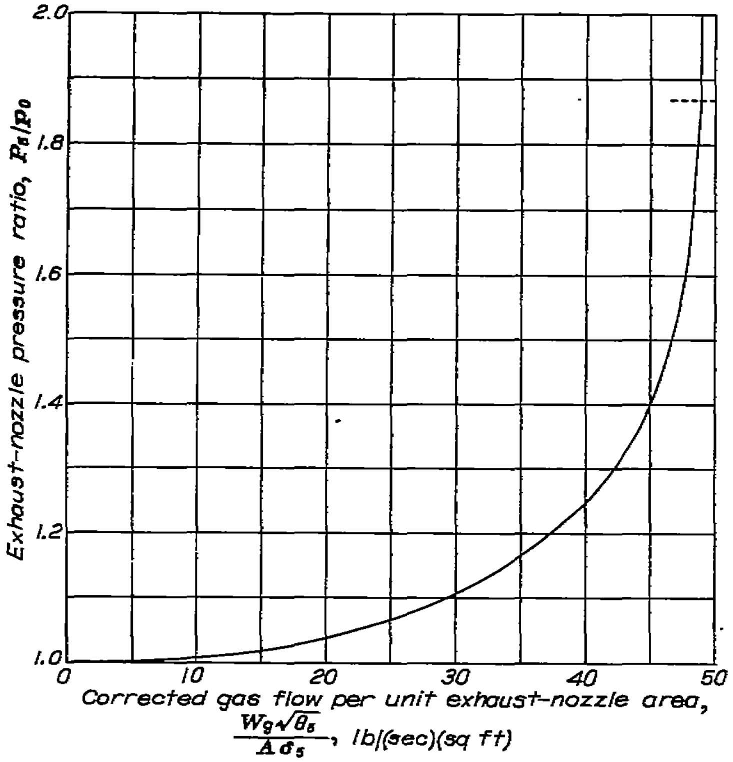


FIGURE 11.—Variation of exhaust-nozzle pressure ratio with gas flow per unit exhaust-nozzle area corrected to total pressure and temperature in the exhaust nozzle relative to NACA standard atmospheric conditions at sea level. (Obtained from unpublished data.)

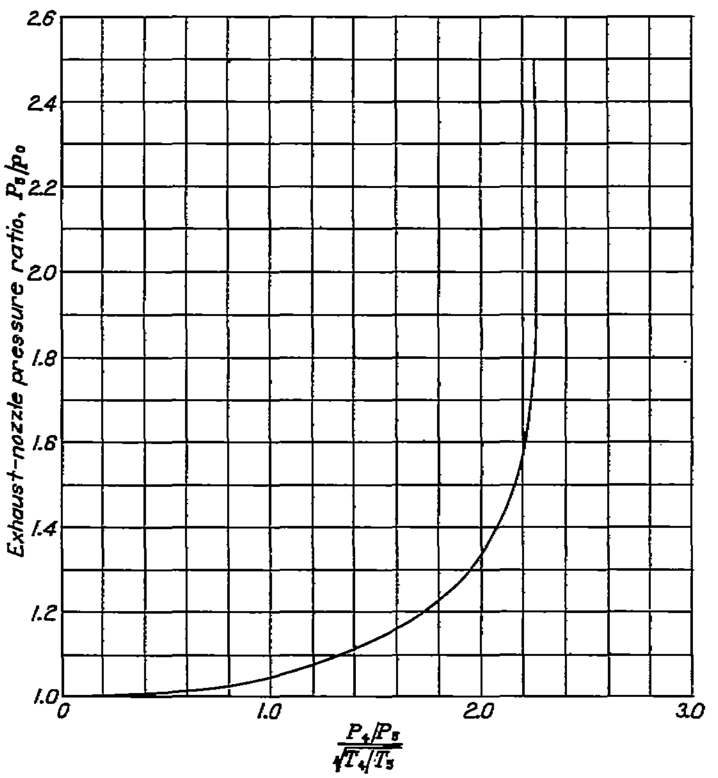


FIGURE 12.—Relation of exhaust-nozzle pressure ratio and turbine total-pressure- and temperature-ratio parameters for exhaust-nozzle area of 1.87 square feet.

From equation (B5) and figure 11, which is plotted from unpublished data, values can be obtained for a plot of P_5/p_0 as a function of $(P_4/P_5)/\sqrt{T_4/T_5}$ for an exhaust-nozzle area of 1.87 square feet. This relation is shown in figure 12.

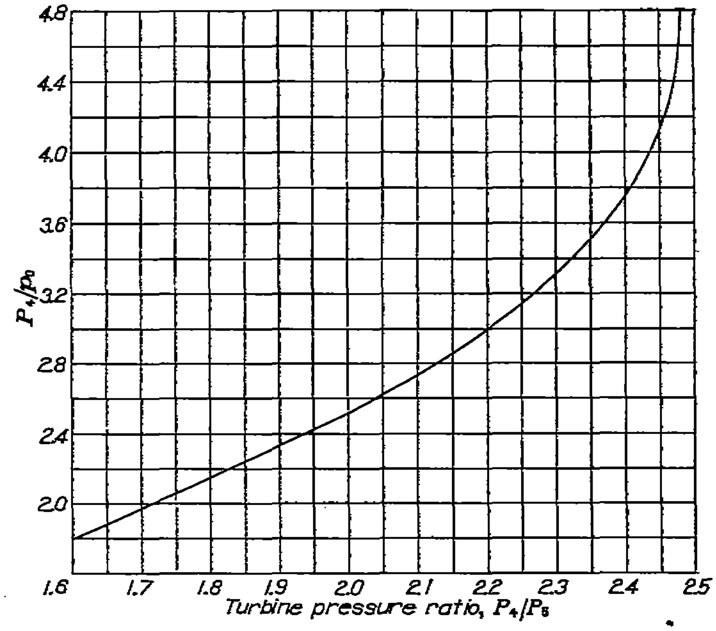


FIGURE 13.—Relation between engine-pressure ratio and turbine-pressure ratio for exhaust-nozzle area of 1.87 square feet.

Figures 10 and 12 can then be used to obtain P_4/p_0 as a function of P_4/P_5. This relation is shown in figure 13.

The turbine torque for each condition for which compressor torque was computed can now be found. In this analysis, it was assumed that P_4/P_3 was constant at 0.95, which determines P_4 for each compressor point, and that W_g/W_a was constant at 1.015, which determines W_g for each compressor point. With the use of the value of P_4 obtained by applying one of these assumed relations to the value of P_3 obtained from the compressor-torque calculations, $\sqrt{T_4/T_5}$ can now be obtained through the use of figures 13 and 10, successively. The turbine torque corresponding to the assumed conditions of ram pressure ratio, ambient conditions, engine speed, and turbine-inlet temperature can then be computed from the following equation:

$$Q_T=c_{p,T}(T_4-T_5)\,\frac{W_g(778)(60)}{2\pi N} \tag{B6}$$

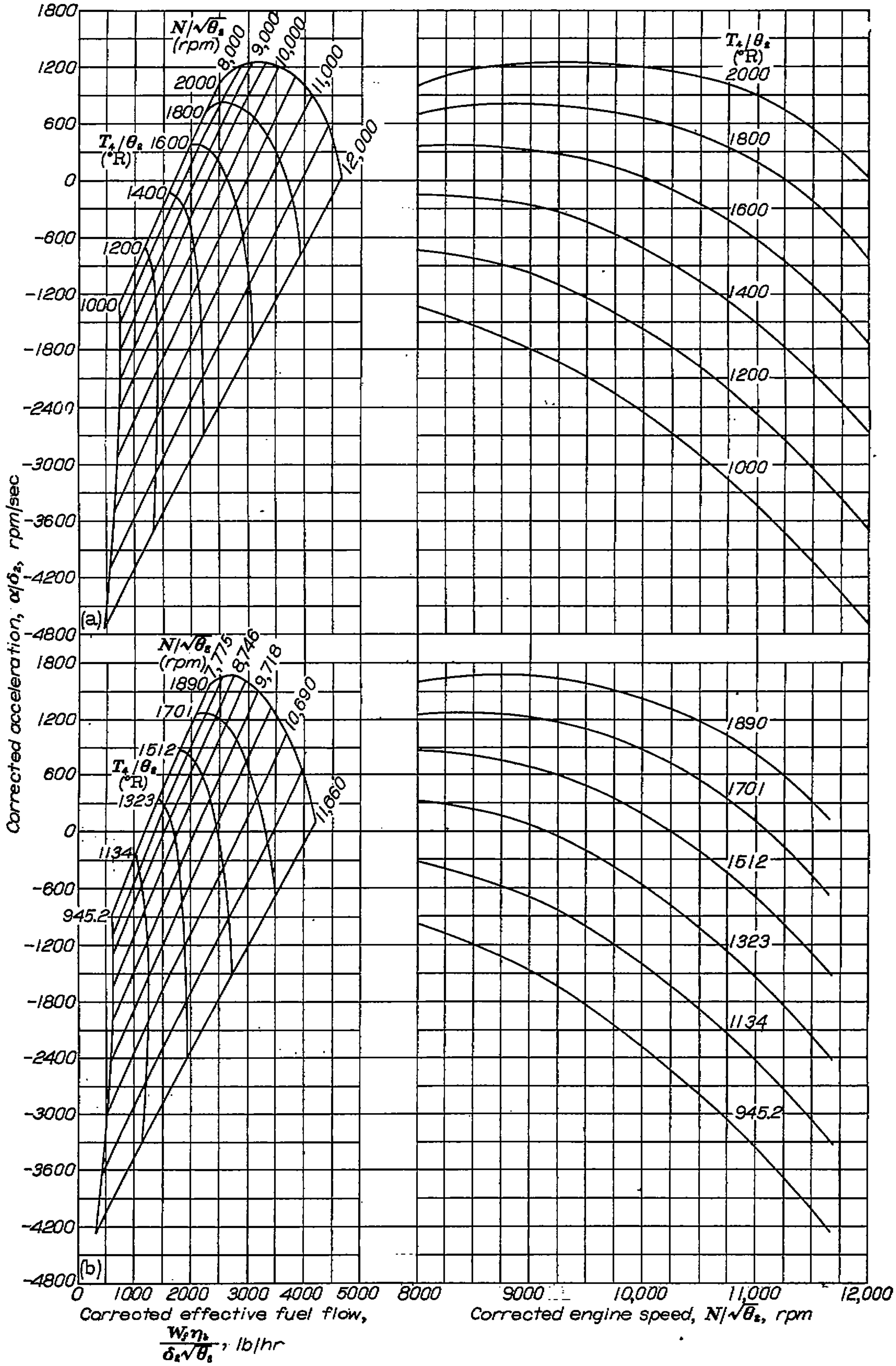
The accelerating torque can then be calculated from the following equation in which 1 percent of the turbine torque or power was assumed lost to the accessories and to gear and bearing friction:

$$Q=0.99\,Q_T-Q_C=I\alpha \tag{B7}$$

A plot of angular acceleration as a function of engine speed, turbine-inlet temperature, and effective fuel flow $W_f\eta_b$ for ram conditions corresponding to an airplane velocity of 0 and 340 miles per hour at sea level, corrected to NACA standard sea-level conditions is shown in figure 14. The effective fuel flows were obtained by use of the following equation, which assumes no time lag between temperature and fuel flow:

$$W_f\eta_b=\frac{W_a(h_4-h_3)\,3600}{H_f} \tag{B8}$$

The values of h_4 and h_3 corresponding to the assumed temperature T_4 and the temperature T_3 calculated from equation (B1) can be found from a table of enthalpies. Figure 3 was obtained by plotting the constant-speed lines of figure 14 and then cross-plotting to obtain the constant-fuel-flow curves.



(a) Ram pressure ratio, 1.0.
(b) Ram pressure ratio, 1.2.

FIGURE 14.—Relation among acceleration, effective fuel flow, turbine-inlet temperature, and engine speed corrected to compressor-inlet conditions relative to NACA standard atmospheric conditions at sea level as calculated from steady-state data.